

# ***Valuing Warrants:*** Dilution and Down-Round Price Protection

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# Valuing Warrants: Dilution and Down-Round Price Protection

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## Introduction

This paper addresses two warrant valuation topics: dilution and down-round price protection. I start with dilution because there is lingering confusion among appraisers on this subject. I address down-round price protection because the SEC has highlighted this issue, noting that a simple Black-Scholes-Merton<sup>1</sup> (BSM) equation may not accurately value the warrants when there is down-round protection, that is, where the exercise price on the warrants will be lowered to match the price of any new financing at a lower price. I illustrate two alternative valuation methods: One combines a lattice and the BSM equation, and the other employs Monte Carlo simulation.<sup>2</sup>

With respect to dilution, I show that it is not an important concern in valuing warrants as long as you use common stock volatility when using a simple BSM equation and equity volatility when using the BSM equation modified for warrants. With respect to down-round protection, I illustrate the magnitude of the effects on the protection, which are facts specific.

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<sup>1</sup>F. Black and M. S. Scholes, "The pricing of options and corporate liabilities," *Journal of Political Economy* 81 (May–June 1973):637–654.

<sup>2</sup>P. Boyle, "Options: A Monte Carlo approach," *Journal of Financial Economics* 4 (1978):323–338.

## Plain Vanilla Warrants and Dilution

Although the valuation of warrants is well documented, there is some confusion about the role of dilution in that valuation. Therefore, the first step is to identify two different dilution effects connected to warrants. I refer to one type as participation dilution and the other as non-fair-value dilution.

The holder of a warrant buys common stock from the issuing firm when the stock is worth more than the warrant holder pays.<sup>3</sup> This transaction reduces or dilutes the proportionate ownership of the existing common stock in the firm's equity appreciation. I refer to this as participation dilution. Firms most frequently create warrants as part of an issuance of bonds or preferred stock with warrants attached. It is reasonable to think of those transactions as being at fair value, with the value of the warrant being embedded in the overall exchange of cash for securities. Because they are at fair value, the values of other securities, in particular, common stock, do not change. Therefore, participation dilution does not typically give rise to any change in the value of the common stock at the time the warrants are issued. Participation dilution can be incorporated in a valuation model by modifying the standard BSM equation.

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<sup>3</sup>Employee stock options are like warrants in this regard. In contrast, market-trade call options do not directly affect the common stock of the firm.

Warrants can also be valued quite accurately using the BSM equation without adjusting for participation dilution. The two models differ in their definitions of the underlying asset and the associated volatility.

Textbooks often introduce a second source of dilution by considering examples in which warrants are distributed for free. Because the firm gives away the warrants, the stock price must decline so that the total value of equity, common stock plus warrants, is unchanged. I refer to this as non-fair-value dilution.<sup>4</sup> I believe that non-fair-value dilution is the source of two misimpressions: (a) Issuing warrants dilutes the common stock and *always* reduces its price, and (b) once the common stock price responds to the issuance of warrants, there is no further dilution effect. The following two examples address those misimpressions.

### Example 1

Consider an all-equity firm with 1.00 million ( $N$ ) shares of common stock priced at \$10.00 ( $S$ ) per share. The firm issues 0.50 million warrants ( $M$ ) to buy one share each at a price of \$10.00 ( $X$ ) per share. The warrants have a five-year term ( $T$ ), the firm pays no dividends, the volatility ( $\sigma$ ) of the firm is 40.00%, and the risk-free rate of interest is 3.00% per year compounded continuously. Following Hull's example, I assume the warrants are distributed for free (see footnote 4). In that case, Hull, and others, have shown that the value of the warrant will be  $N/(N + M)$ ,  $[1.00/(1.00 + 0.50)]$  in this example, multiplied by the value of a BSM call option with  $S = \$10.00$ ,  $X = \$10.00$ ,  $r = 3.00\%$ ,  $T = 5.00$ , and  $\sigma = 40.00\%$ . In this example, the value of the warrant ( $W$ ) is:

$$W = (1.00/1.50)(\$3.9508) = \$2.6339.$$

The warrants in aggregate are worth \$1.3169 million. Therefore, the common stock must be worth \$8.6831 million or \$8.6831 per share. The value of the common stock falls from \$10.00 to \$8.6831 when the warrant issuance is announced. Again, for clarity, I refer to this as the non-fair-value dilution effect.

Having taken into consideration the non-fair-value effect, can we ignore any other dilution effect in valuing warrants? In general, we cannot. To see why, consider the valuation of this warrant after the stock trades at \$8.6831. If we value the warrant as a call option with the parameters  $S = \$8.6831$ ,  $X = \$10.00$ ,  $r = 3.00\%$ ,  $T = 5.00$ , and  $\sigma = 40.00\%$ , its value is \$3.0231. This is 14.78% higher than the value already calculated. This error has two sources. By far the more important of the

two sources of error is our use of an incorrect volatility. We previously calculated the value of a call option on the total firm, common stock plus warrants; we are now calculating the value of a call option on the common stock only. Because the warrants are more volatile than the common stock, their volatility must be greater than 40.00%, and the volatility of the common stock must be less than 40.00%. By using 40.00% volatility in our valuation of the warrant as a call option on the common stock, we overestimated the value of the warrant. I elaborate on this relationship in example 2.

### Example 2

The parameters of this example are the same as in example 1, but instead of giving away the warrants, we sell them for fair value. Our challenge is to determine the value of the warrants. Issuing warrants for cash increases the value of the firm. The value of the firm per share of common stock is  $\$10.00 + WM/N$ . With that change, the warrant can be valued as  $N/(N + M)$  fraction of a call option with  $S = \$10.00 + W(0.5/1.00)$ ,  $X = \$10.00$ ,  $r = 3.00\%$ ,  $T = 5.00$ , and  $\sigma = 40.00\%$ . Note that we have a small challenge in that the value of the warrant,  $W$ , is defined in terms of a stock price that includes  $W$ . We need to use a search process to solve one equation written in terms of one unknown,  $W$ . The solution is a value per warrant of \$3.5280.<sup>5</sup> In this example, we eliminated the non-fair-value dilution effect but continue to include the participation-dilution effect. I now explore in more detail the size of the participation-dilution effect.

In example 1, there was a substantial error when we ignored the dilution effect and valued the warrant as a call option on the common stock. That error had two sources. One was the use of an inappropriate volatility, and the other was the omission of the participation-dilution effect. I extend this example to partition those two effects and document their relative magnitudes. To do that, the volatility of the common stock needs to be identified in a capital structure composed of common stock and warrants on the common stock.

Whaley details the relationships that address the valuation of warrants and other securities.<sup>6</sup> The volatility of the common stock,  $\sigma_S$ , is related to (a) the volatility of total equity,  $\sigma_E$ , (b) the delta of the common stock,  $N(d_1)$ ,

<sup>5</sup>Note that if we apply this same approach to example 1 after the warrants have been distributed, we correctly value the warrants.  $S = \$8.6831 + W(0.5/1.50)$ ,  $X = \$10.00$ ,  $r = 3.00\%$ ,  $T = 5.00$ , and  $\sigma = 40.00\%$  provides a value for the warrant of \$2.6339, and the value of  $S = \$8.6831 + W(0.5/1.50) = \$10.00$  is not surprising.

<sup>6</sup>Robert E. Whaley, *Derivatives Markets, Valuation and Risk Management* (Hoboken, New Jersey: John Wiley & Sons Inc., 2006), chapter 12, 439–444.

<sup>4</sup>See, for example, John Hull, *Options, Futures and Other Derivatives*, 6th ed. (Upper Saddle River, New Jersey: Prentice Hall, 2006), 298–300.

**Table 1**

Comparison of Warrant Valuations  $E = \$10$  Million,  $N = 1.0$  Million,  $X = \$10$ ,  $r = 3\%$ ,  $T = 5.0$  Years, and  $\sigma_E = 40\%$

Dilution Factor	Total Equity as the Underlying Security		Common Stock as the Underlying Security with Common Volatility		Common Stock as the Underlying Security with Equity Volatility	
	Warrant Value	Stock Volatility	Warrant Value	Error	Warrant Value	Error
4.8%	\$3.8990	39.3%	\$3.9025	0.09%	\$3.9508	1.33%
9.1%	\$3.8498	38.7%	\$3.8567	0.18%	\$3.9508	2.62%
20.0%	\$3.7158	37.0%	\$3.7316	0.43%	\$3.9508	6.33%
33.3%	\$3.5280	34.7%	\$3.5560	0.80%	\$3.9508	11.99%
50.0%	\$3.2414	31.2%	\$3.2870	1.41%	\$3.9508	21.89%

and (c) the ratio of the value of the equity ( $E$ ) to the value of the common stock ( $S$ ):

$$\sigma_S = N(d_1)(E/S)\sigma_E.$$

The delta of the common stock is equal to one minus the delta of the warrant, which is equal to  $M/(N + M)$  multiplied by the value of  $N(d_1)$  calculated in a call option formula with the underlying asset equal to total equity,  $E = \$10.00 + W(0.5/1.0)$ ,  $X = \$10.00$ ,  $r = 4.00\%$ ,  $T = 5.00$ , and  $\sigma = 40.00\%$ . Specifically,

$$\begin{aligned}\sigma_S &= [1 - N(d_1)M/(N + M)](E/S)\sigma_E \\ &= [1 - N(0.7966)(0.5/1.5)](11.7640/10.00)40.00\% \\ &= 0.7376(11.7640/10.00)40.00\% \\ &= 34.71\%.\end{aligned}$$

If we calculate the value of the warrant with this volatility and ignore the participation-dilution effect, we have  $S = \$10.00$ ,  $X = \$10.00$ ,  $r = 3.00\%$ ,  $T = 5.00$ ,  $\sigma = 34.71\%$ , and  $W = \$3.5560$ . This is only 0.80% larger than the earlier estimate of \$3.5280. If we use a volatility of 40.00%, the warrant value is \$3.9508, a difference of 11.99%. Therefore, the vast majority of pricing error is attributable to failing to match the underlying asset and the volatility, as opposed to failing to model the participation-dilution effect. Table 1 expands this result for example 2 using a range of dilution factors,  $M/(N + M)$ , from 5% to 50%. The participation-dilution error is the difference between (a) the warrant value when you use total equity as the underlying security, total equity volatility, and you consider dilution, and (b) the warrant value when you use common stock as the underlying asset, common stock volatility, and you ignore dilution. This error is relatively small, ranging from 0.09% to 1.41%.<sup>7</sup> The error when you use common stock as the

underlying asset, total equity volatility, and ignore dilution is relatively large, ranging from 1.33% to 21.89%.

In summary, the only important dilution that changes the share price of a company's stock occurs when warrants are issued and the company receives less than their fair value. I believe this rarely occurs. Both outstanding warrants and to-be-issued warrants can be valued accurately using either the standard BSM formula or the BSM formula modified to incorporate the effect of upside participation by the warrants. The standard BSM formula requires an estimate of the volatility of the common stock, which is more readily available from trading data when the warrants are already outstanding. The modified BSM formula requires an estimate of the volatility of the common stock and warrants combined, which is more readily available from trading before warrants have been issued.

### Warrants with Down-Round Protection

When warrants are created as part of a package of securities that finance early stage private companies, they sometimes include down-round protection. A typical reset provision would lower the strike price of the warrant in the event that the firm issues securities at a lower price in the future. To limit the complexity of the discussion, I will consider the valuation of warrants on common stock. Moreover, I will use the perspective about dilution developed in the previous section; namely, I will value the warrants as call options on the common stock, keeping in mind that the volatility I use is the common stock volatility.

A popular approach to valuing warrants with reset provisions is to ignore the reset feature and apply the BSM formula. One justification for ignoring the reset provision is the claim that the probability of issuing securities at a lower price is zero. This view flies in the face of the decision of the contracting parties to include a reset provision: If it can never happen, why is it part of the contract? A modified view is that the probability is

<sup>7</sup>This is consistent with research by Jacob Sidenius, "Warrant pricing—Is dilution a delusion," *Financial Analysts Journal* (September/October 1996):77–80.

**Table 2A**

An Even  $n$ -Step Valuation Lattice for a Warrant with a Single Reset after  $\alpha T$  Years  $\Delta t = \alpha T / n$ ,  $u = e^{\sigma \sqrt{\Delta t}}$ ,  $d = 1/u$ , and  $p = (r\Delta t - d)/(u - d)$

0	$\Delta t$	$2\Delta t$	$3\Delta t$	$\alpha T$		Payoff at Date $\alpha T$
S	$Su$ $Sd$	$Su^2$ $Sud$ $Sd^2$ $Sd^3$	$Su^2$ $Sud$ $Sud^2$ $Sd^3$	$Su^n$ $Su^{n-1}d$ $\dots$ $Su^{n/2}d^{n/2}$ $Su^{n/2-1}d^{n/2+1}$ $\dots$ $Sd^n$	$\dots$ $\dots$ $\dots$ $\dots$ $\dots$ $\dots$ $\dots$	BSM( $Su^n$ , min( $X, Su^n$ ), $r$ , $(1 - \alpha)T$ , $\sigma$ )
						BSM( $Su^{n-1}d$ , min( $X, Su^{n-1}d$ ), $r$ , $(1 - \alpha)T$ , $\sigma$ )
						BSM( $Su^{n/2}d^{n/2}$ , min( $X, Su^{n/2}d^{n/2}$ ), $r$ , $(1 - \alpha)T$ , $\sigma$ )
						BSM( $Su^{n/2-1}d^{n/2+1}$ , min( $X, Su^{n/2-1}d^{n/2+1}$ ), $r$ , $(1 - \alpha)T$ , $\sigma$ )
						BSM( $Sd^n$ , min( $X, Sd^n$ ), $r$ , $(1 - \alpha)T$ , $\sigma$ )
						BSM( $Su^{n/2}d^{n/2}$ , min( $X, Su^{n/2}d^{n/2}$ ), $r$ , $(1 - \alpha)T$ , $\sigma$ )
						BSM( $Su^{n/2-1}d^{n/2+1}$ , min( $X, Su^{n/2-1}d^{n/2+1}$ ), $r$ , $(1 - \alpha)T$ , $\sigma$ )
						BSM( $Su^{n/2}d^{n/2}$ , min( $X, Su^{n/2}d^{n/2}$ ), $r$ , $(1 - \alpha)T$ , $\sigma$ )
						BSM( $Su^{n/2-1}d^{n/2+1}$ , min( $X, Su^{n/2-1}d^{n/2+1}$ ), $r$ , $(1 - \alpha)T$ , $\sigma$ )
						BSM( $Sd^n$ , min( $X, Sd^n$ ), $r$ , $(1 - \alpha)T$ , $\sigma$ )

sufficiently low that the effect of the reset provision on the value of the warrant is immaterially small. That may often be the case. It depends on both the probability of a security issuance and the security characteristics that determine the value of a reset when it occurs. My objective is to provide evidence relevant to this assertion.

I consider two approaches to valuing reset provisions, one relatively straightforward and one more complex. The former allows for a reset event at only one date. It requires the creation of a lattice and the application of the BSM formula. The latter allows resets at multiple dates and incorporates a probability of reset at each date.

For a single reset date, the warrant and its reset provision can be valued using a lattice and the BSM formula as shown in the Table 2A. This presentation assumes a standard Cox-Ross-Rubinstein<sup>8</sup> lattice cover-

ing the period from the valuation date to the forecast reset date. In this illustration, I assume that the warrant's exercise price resets based on a new issuance of common stock after  $\alpha T$  years, where  $T$  is the original time to expiration of the warrant. To make the formulas general, I assume an even number of steps in the lattice. The expression BSM( $S, X, r, \sigma, T$ ) represents the BSM value of a call option for the associated variables. The payoffs at the end nodes of the lattice are the BSM option values of call options with a remaining life of  $(1 - \alpha)T$  years. The call option values are discounted back through the lattice to determine the value of the warrant. Where the stock prices of the end nodes in the lattice are above  $X$ , the exercise price in the call option calculation is the original exercise price of  $X$ . Where the end-node stock prices are less than  $X$ , the exercise prices in the option calculations are those lower stock prices.

Table 2B provides values that match Table 2A for an example in which a firm with a common stock price of

**Table 2B**

Valuation Lattice for a Warrant with a Single Reset after  $\alpha T$  Years  $S = \$10.00$ ,  $X = \$10.00$ ,  $r = 2.0\%$ ,  $\sigma = 50\%$ ,  $T = 8.0$ ,  $\alpha = 0.5$ ,  $n = 100$

0.00	0.04	0.08	0.12	4.00	Payoff at Date 4.00
\$10.00	\$11.05 \$9.05	\$12.21 \$10.00 \$8.19	\$13.50 \$11.05 \$9.05 \$7.41	$\dots$ $\dots$ $\dots$ $\dots$ $\dots$ $\dots$ $\dots$ $\dots$ $\dots$ $\dots$	\$220,265 \$180,337 $\dots$ $\dots$ \$10.00 \$8.19 $\dots$ $\dots$ \$0.00045 \$0.00019



**Table 3**

Value of a Warrant with a Single Reset after 4.00 Years  $S = \$10.00$ ,  $X = \$10.00$ ,  $r = 2.0\%$ ,  $\sigma = 50\%$ ,  $T = 8.0$

Volatility	Fraction of Time to Reset								
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
	Warrant Value								
10%	\$2.08	\$2.11	\$2.12	\$2.12	\$2.11	\$2.10	\$2.08	\$2.06	\$2.02
20%	\$3.11	\$3.16	\$3.19	\$3.19	\$3.19	\$3.18	\$3.15	\$3.12	\$3.06
30%	\$4.12	\$4.18	\$4.22	\$4.23	\$4.23	\$4.22	\$4.19	\$4.15	\$4.08
40%	\$5.05	\$5.13	\$5.17	\$5.18	\$5.18	\$5.17	\$5.15	\$5.10	\$5.02
50%	\$5.90	\$5.98	\$6.02	\$6.04	\$6.04	\$6.03	\$6.00	\$5.96	\$5.88
60%	\$6.66	\$6.73	\$6.77	\$6.78	\$6.79	\$6.78	\$6.75	\$6.71	\$6.63
70%	\$7.31	\$7.38	\$7.42	\$7.43	\$7.43	\$7.43	\$7.40	\$7.36	\$7.29
80%	\$7.88	\$7.93	\$7.96	\$7.98	\$7.98	\$7.97	\$7.95	\$7.91	\$7.84
90%	\$8.35	\$8.40	\$8.42	\$8.43	\$8.43	\$8.42	\$8.40	\$8.36	\$8.29
	Increase in Value Created by the Reset Feature								
10%	7.2%	8.6%	9.0%	8.9%	8.6%	8.0%	7.1%	5.9%	4.3%
20%	7.9%	9.7%	10.6%	10.9%	10.8%	10.3%	9.5%	8.2%	6.2%
30%	7.4%	9.1%	10.0%	10.4%	10.4%	10.0%	9.4%	8.2%	6.3%
40%	6.5%	8.1%	8.9%	9.3%	9.3%	9.1%	8.5%	7.6%	5.9%
50%	5.6%	7.0%	7.7%	8.1%	8.1%	7.9%	7.5%	6.6%	5.2%
60%	4.8%	6.0%	6.6%	6.8%	6.9%	6.7%	6.4%	5.7%	4.5%
70%	4.0%	5.0%	5.5%	5.7%	5.7%	5.6%	5.3%	4.7%	3.7%
80%	3.3%	4.1%	4.5%	4.7%	4.7%	4.6%	4.3%	3.8%	2.9%
90%	2.7%	3.3%	3.6%	3.8%	3.7%	3.6%	3.3%	2.9%	2.0%

\$10.00 has issued eight-year warrants with an exercise price of \$10.00. I assume a common stock issuance occurs after 4.00 years, and the exercise price of the warrants is lowered if the stock price has declined. The risk-free rate of interest is 2% compounded continuously, and the volatility of the common stock is 50%. The number of steps in the lattice is 100. The two top nodes in the lattice represent extremely small probability outcomes with extremely high warrant values and no reset. The third node shown represents the case of an equal number of up and down moves in the lattice and a stock price after 4.00 years equal to the starting value, \$10.00. In that case, the exercise price is not reset, and the value of a four-year warrant is \$4.08. The next node represents the case of forty-nine up and fifty-one down moves with a resulting stock price after 4.00 years of \$8.19. If the exercise price on the warrant were not reset, the value of the warrant would be \$2.84. With the exercise price reset to \$8.19, the value is \$3.34. Each of the lattice outcomes in the lower half of the lattice produces a higher warrant value than would be the case without the reset feature. It is worth noting that for all of the prices \$10.00 and lower, the payoff in the lattice is that of an at-the-money warrant, which in this example is worth 40.8% of the common stock price. Taking into account the reset feature, the value of the warrant when issued is \$6.04, which is 8.1% higher than the value of a similar warrant without a reset feature, \$5.59.

Table 3 reports the values of warrants with the same basic features and a range of common stock volatilities and times to reset. On a percentage basis, the reset feature has a maximum effect for volatilities in the range of 20% to 30% but is smaller for both higher and lower volatilities. The maximum percentage effect occurs when the reset is approximately halfway through the life of the warrant. If the reset occurs very early, the potential for downward movement in the exercise price is smaller; if the reset occurs very late, you have a lower time value of the option associated with the lower reset. In this example, the value of the reset feature is less than 11% of the value of the warrant. Suppose the probability of a security issuance is 20%. In that case, the value of the reset provision relative to the value of the warrant is likely to be less than 2.2% [(11%)(20%)]. While each set of circumstances will differ, this analysis suggests that ignoring the effect on value of warrant reset provisions may be reasonable in many instances.

If we use a Monte Carlo simulation analysis, we can introduce more complex, and perhaps more realistic, financing and reset scenarios. To illustrate the Monte Carlo approach and the potential effects of a more complex set of financing possibilities, I consider new issuance at the end of each of the first seven years. I treat the decision to issue new securities as a random variable with a constant probability of occurrence each year. The

**Table 4**

Value of a Warrant with a Probability of Reset Each Year  $S = \$10.00$ ,  $X = \$10.00$ ,  $r = 2.0\%$ ,  $\sigma = 50\%$ ,  $T = 8.0$

Volatility	Probability of Annual Reset						
	0.14	0.29	0.43	0.57	0.71	0.86	1.00
10%	\$2.06	\$2.16	\$2.23	\$2.28	\$2.32	\$2.35	\$2.38
20%	\$3.10	\$3.27	\$3.40	\$3.49	\$3.56	\$3.62	\$3.67
30%	\$4.12	\$4.33	\$4.49	\$4.60	\$4.70	\$4.77	\$4.83
40%	\$5.07	\$5.30	\$5.47	\$5.60	\$5.70	\$5.78	\$5.84
50%	\$5.94	\$6.18	\$6.35	\$6.48	\$6.57	\$6.65	\$6.71
60%	\$6.74	\$6.96	\$7.13	\$7.25	\$7.34	\$7.41	\$7.47
70%	\$7.48	\$7.69	\$7.83	\$7.94	\$8.02	\$8.09	\$8.14
80%	\$8.19	\$8.37	\$8.50	\$8.59	\$8.66	\$8.72	\$8.77
90%	\$8.91	\$9.06	\$9.17	\$9.25	\$9.31	\$9.36	\$9.40
Increase in Value Created by the Reset Feature							
10%	6.3%	11.0%	14.6%	17.3%	19.5%	21.3%	22.7%
20%	7.8%	13.5%	17.9%	21.1%	23.6%	25.7%	27.4%
30%	7.5%	13.0%	17.1%	20.1%	22.5%	24.4%	26.0%
40%	6.9%	11.8%	15.4%	18.1%	20.1%	21.8%	23.2%
50%	6.4%	10.6%	13.7%	15.9%	17.7%	19.1%	20.2%
60%	6.2%	9.7%	12.2%	14.1%	15.6%	16.7%	17.7%
70%	6.4%	9.3%	11.4%	12.9%	14.1%	15.1%	15.8%
80%	7.4%	9.8%	11.5%	12.7%	13.7%	14.4%	15.0%
90%	9.6%	11.5%	12.8%	13.8%	14.5%	15.1%	15.6%

issuance is independent of the stock price and of any previous financing. In a real-life situation, it may be appropriate to change both of these assumptions. Table 4 reports the values of the warrants with a reset feature for ranges of probabilities of new issuances and volatilities. The table also reports the percentage increase in value attributable to the reset feature. I selected the probabilities 0.14, 0.29, ..., 1.0 (1/7, 2/7, ..., 7/7) such that the expected number of times a new financing occurs is 1, 2, ..., 7. It seems reasonable to focus on the results for new financing one to three times. For one financing, the value increases range from 6.3% to 9.6%. These values are similar to those shown in Table 3, where the average value across all combinations of a single financing at different points in the eight-year period is 6.6%. When the expected number of financings is two, the value of the reset provision ranges from 9.3% to 13.5%; when it is three, the range is 11.4% to 17.9%. These indicative values may be useful in determining when a reset feature should be valued and when it can reasonably be ignored.

## Summary

There are two types of dilution that play a role in warrant valuation. Non-fair-value dilution occurs when warrants are distributed for less than their fair value. In this case, the value of the common stock decreases, and it is important to take that into consideration when valuing warrants. It is also reasonable to believe that this case is

primarily an artifact of textbook examples. Participation dilution occurs because warrants share the appreciation of the firm's value. This dilution can be modeled, or not, because its effect is relatively small. However, it is important to appreciate that when modeled, the volatility used should be the volatility of the common stock and warrants, and when this dilution is not modeled, the volatility should be that of the common stock only.

I illustrated two methods to value warrants with a reset feature. One method assumes that there is a single future financing at a known date and requires the combination of a lattice and the BSM formula. For an eight-year warrant, the reset feature increased the warrant value by 2% to 11%, depending on the stock volatility and the timing of the new financings. If the probability of a new financing is small, for example 20%, then the overall influence on value is likely to be small enough to ignore, i.e., 0.4% to 2.2% of the value of the warrant, which is itself typically a small percentage of the capital structure. The second method employs Monte Carlo simulation and allows for multiple financings at random times over the life of the warrant. As the expected value of the number of times the firm issues common stock increases, the value of the reset feature increases. When the expected number of future issuances is three over an eight-year period, the reset feature adds between 11% and 18% to the value of the warrant. Whether this is large enough to require inclusion in a valuation will depend on the size of the warrant position.

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## *About the author*

**Dwight Grant** is a managing director in PwC's Value Consulting group, based in the San Francisco office. He advises clients on complex financial instruments and oversees professional standards and technical training for the practice.

PwC's Value Consulting group advises clients on the analysis and valuation of complex financial instruments, such as derivatives, hybrid securities, hedging instruments and contingent payment agreements. Services primarily focus on valuations and measuring the fair value of assets for financial reporting, tax planning, transactions and decision-making purposes.

Dwight has over 30 years of experience providing valuation services to clients. He previously taught at Thunderbird, The Garvin School of International Management and was also a visiting professor at Fundação Armando Alvares Penteado in São Paulo, Brazil. He holds a BA in economics from University of Western Ontario, an MBA in finance from Wharton at University of Pennsylvania and a Ph.D. in finance from the University of Pennsylvania.

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